

ATMS 380 Homework #1, due Wednesday, Jan 12, 2011

For additional instructions, see

<http://www.atmos.washington.edu/academics/classes/2011Q1/380/HW1.html>

The one-dimensional advection equation:

$$\frac{\partial \Psi}{\partial t} + c \frac{\partial \Psi}{\partial x} = 0$$

with constant wind speed c has exact solution $\Psi(x, t) = \Psi(x - ct, 0)$.

Your task is to write down the finite difference model of this advection equation with centered space difference and forward time difference. You will code it in matlab and vary some parameters to investigate the behavior. A matlab script is provided for you at the web address above with leapfrog time differencing already coded:

$$\frac{\Psi_j^{n+1} - \Psi_j^{n-1}}{2\Delta t} + c \frac{\Psi_{j+1}^n - \Psi_{j-1}^n}{2\Delta x} = 0$$

and an upstream differencing already coded

$$\frac{\Psi_j^{n+1} - \Psi_j^n}{\Delta t} + c \frac{\Psi_j^n - \Psi_{j-1}^n}{\Delta x} = 0.$$

Let the initial condition be $\Psi(x, 0) = \sin^6(2\pi x)$ on the periodic domain $0 \leq x \leq 1$, where for convenience x has been nondimensionalized. Initially the wind speed c is 0.1 (also nondimensionalized). The Courant number $c\Delta t/\Delta x$ is also initially set to 0.1. See the web address for questions to answer about this problem.

Using nondimensionalized variables is just a convenience and is typical in models. For example, if the length scale of interest is 1000 m, the wind speed would be 100 m/s and the domain would be 1000 m wide.