Contents

CHAPTER 1

Interpretation of zonal- and time-averaged second moment terms

1. Basic Definitions

We will be concerned with terms of the form $[\overline{xy}]$, where x any y represent any scalar meteorological parameters (e.g., u, v, T, Φ,etc.). The overbar and the brackets define the time and zonal averaging operators; i.e.,

$$\overline{x} = \frac{1}{T}\int_0^T x dt \tag{1.1}$$

$$[x] = \frac{1}{2\pi}\oint x d\lambda \tag{1.2}$$

where t is time and λ is longitude. Departures from a time average

$$x' = x - \overline{x} \tag{1.3}$$

will be referred to as transients and departures from a zonal average

$$x^* = x - [x] \tag{1.4}$$

will be referred to as eddies. Note that by definition

$$\overline{x'} = 0 \tag{1.5}$$

and

$$[x^*] = 0. \tag{1.6}$$

Double averaging operators are redundant, i.e.,

$$\overline{\overline{x}} = \overline{x} \tag{1.7}$$

and

$$[[x]] = [x] \tag{1.8}$$

The time and zonal averaging operators are commutative, i.e.,

$$\overline{[x]} = [\overline{x}]\,. \tag{1.9}$$

In order to visualize these identities, one can imagine x as representing a two-dimensional "observation matrix" x_{ij} where the index i refers to the longitude of a particular observation and j to its order in the time sequence. The terms in (1.9) represent averages over all the elements in the matrix. The order of the averaging (i.e., whether by rows first or by columns first) is irrelevant.

2. Decomposition of product terms

Time averages of products can be expanded in the form

$$\begin{aligned}\overline{xy} &= \overline{(\overline{x}+x')(\overline{y}+y')} \\ &= \overline{\overline{x}\overline{y}+x'y'+x'\overline{y}+\overline{x}y'}\end{aligned}$$

Since $\overline{x'\overline{y}} = 0$ and $\overline{\overline{x}y'} = 0$ and the double averaging operator on the first term is redundant, it follows that

$$\overline{xy} = \overline{x}\ \overline{y} + \overline{x'y'} \tag{1.10}$$

where the first yerm on the right hand side can be identified with the time mean and the second term with the transients. In a similar manner it can be shown that

$$[xy] = [x]\,[y] + [x^* y^*] \tag{1.11}$$

The zonal mean of (1.10)

$$[\overline{xy}] = [\overline{x}\ \overline{y}] + \left[\overline{x'y'}\right]$$

can be expanded by writing $\overline{x} = [\overline{x}] + \overline{x}^*$and $\overline{y} = [\overline{y}] + \overline{y}^*$ in the first term on the right hand side. After simplifying the resulting equation, we obtain

$$[\overline{xy}] = [\overline{x}]\,[\overline{y}] + [\overline{x}^*\overline{y}^*] + \left[\overline{x'y'}\right] \tag{1.12}$$

In this formulation, first derived by Priestly (1949) the first term is identified with the zonally averaged, time mean circulation and resolves features such as the climatological-mean Hadley cell. The second term is identified with eddies that are steady in time and resolves features such as the monsoons. These features have come to be referred to as "stationary waves" although the term "steady eddies" would be more consistent with the nomenclature used here.The final term gives the total contribution of the transients.

In an analogous manner, (1.11) can be time averaged and the first term on the right hand side can be expanded in terms of time means and transients and simplified to obtain

$$\overline{[xy]} = \overline{[x]}\,\overline{[y]} + \overline{[x]'\,[y]'} + \overline{[x^* y^*]} \tag{1.13}$$

In this formulation, first derived by Starr and White (1951) the first term on the right hand side is identical to the corresponding term in (1.12). The second term is the contribution from the transient zonally symmetric circulations and the third term is the contribution from the eddies. Note that (1.13) can be derived directly from(1.12) simply by replacing all zonal averages and departures from zonal averages by time averages and departures from time averages, or alternatively, (1.12) could be derived from (1.13) by the reverse procedure.

The relationship between(1.12) and (1.13) can be clarified by expanding $[\overline{xy}]$ as follows. Following Lorenz (1953) we begin by writing

$$x = [\overline{x}] + \overline{x}^* + [x]' + x'^*$$

and

$$y = [y] + \overline{y}^* + [y]' + y'^*$$

where the first term on the right hand side refers to the time mean, zonal mean contribution, the second term to the zonally varying, time mean contribution, the third term to the time-varying, zonal-mean contribution and the fourth term to the *transient eddy* contribution. After taking the product and averaging over longitude and time, we obtain, after simplification

$$\overline{[xy]} = \overline{[x]}\,\overline{[y]} + \overline{[x]'\,[y]'} + [\overline{x}^*\overline{y}^*] + [\overline{x'^*y'^*}] \tag{1.14}$$

Note that the first three terms on the right hand side of (1.14) are identical to the terms in (1.12) and/or (1.13).The remaining term is the contribution of the transient eddies. By combining these three equations in various ways it is evident that

$$[\overline{x'y'}] = \overline{[x]'\,[y]'} + [\overline{x'^*y'^*}] \tag{1.14}$$

and

$$\overline{[x^*y^*]} = [\overline{x}^*\overline{y}^*] + [\overline{x'^*y'^*}] \tag{1.15}$$

A convenient way of summarizing these relationships is to summarize them in the matrix

	Zonally symmetric	Eddy	Σ
Steady	$\overline{[x]}\,\overline{[y]}$	$[\overline{x}^*\overline{y}^*]$	$\overline{[x]}\,\overline{[y]}$
Transient	$\overline{[x]'\,[y]'}$	$[\overline{x'^*y'^*}]$	$[\overline{x'y'}]$
Σ	$\overline{[x]\,[y]}$	$\overline{[x^*y^*]}$	$\overline{[xy]}$

3. Statistical interpretation

With the exception of the $\overline{[x]}\,\overline{[y]}$ term all the terms in all the above expansions involve covariances. For example, $[\overline{x'y'}]$ is the temporal covariance between x and y and $[\overline{x}^*\overline{y}^*]$ is the longitudinal covariance between $\overline{x}$ and $\overline{y}$.

The covariance between x and y can be expressed as the product of the correlation coefficient between x and y and the product of the standard deviations of x and y. For example, the temporal covariance is given by

$$x'y' = r(x,y)\sqrt{\overline{x'^2}}\sqrt{\overline{y'^2}} \tag{1.17}$$

amd the longitudinal covariance between $\overline{x}$ and $\overline{y}$ is given by

$$[\overline{x}^*\overline{y}^*] = r(\overline{x}^*,\overline{y}^*)\sqrt{\overline{x}^{*2}}\sqrt{\overline{y}^{*2}} \tag{1.18}$$

The standard deviations are the same as the r.m.s. amplitudes which can often be estimated on the basis of inspection of time series or maps. The correlation coefficient r ranges from -1 to$+1$. High values of r imply a strong linear dependence of one upon the other. The square of the correlation coefficient r^2 is the fraction of the variance of one variable that can be explained based on a knowledge of the other variable. The correlation coefficient r is also the slope of the least squares best fit regression line on a scatter plot of standardized values of the two variables.

4. Calculation of general circulation statistics

In studies of the general circulation x often corresponds to a scalar variable such as zonal momentum per unit mass u or relative vorticity ζ and y to the meridional wind component v or vertical velocity ω. Hence the product of the two corresponds to the poleward or vertical flux of a scalar. For the special case $x = y$, the product corresponds to the variance of x. In studies that were carried out prior to the 1970s, the calculations were generally based on station data. In many cases some form of manual or objective analysis was performed to interpolate the station data onto a field of regularly spaced gridpoints. For example, the studies of Oort and Ramusson (1971) and Newell et al. (1972) relied on objective analysis of fields such as $\overline{u}$, $\overline{v}$, $\overline{T}$, $\overline{u\prime v\prime}$, $\overline{v\prime T\prime}$, etc., where all the time-averaged quantities had been computed on the basis of data from the global network of radiosonde stations. Fields analyzed in this manner tend to be bland and featureless in data-sparse regions because there is no way of introducing information into the gaps between stations. Starting in the late 1970s, gridded fields generated by the data assimilation systems used to initialze operational numerical weather prediction schemes began to be used extensively in general circulation studies. For a discussion of these datasets, see the Appendix for Chapter 8 on the companion web site for the second edition of Wallace and Hobbs.

5. References

Lorenz, E.N., 1953: A multiple index notation for describing atmospheric transport processes. AFCRL Report 53-35.

Newell, R.E., J.W. Kidson, D.G. Vincent and G.J. Boer,1972-1974: The general circulation of the tropical atmosphere and the interactions with extratropical latitudes. The M.I.T. Press, Cambridge, MA, 258 pp.

Oort, A. H. and E. M. Rasmusson,1971: Atmospheric circulation statistics. NOAA, U.S. Dep't of Commerce, 323 pp.

Priestly, C.H.B., 1949: Heat transport and zonal stress between latitudes. Ruart. J. R. Met. Soc., 75, 28-40.

Reynolds, O., 1894: On the dynamical theory of incompressible viscous fluids and the determination of the criteria. Phil. Trans. Roy. Soc., (A) 186: 123-164

tarr, V.P. and R.M. White, 1951: A hemispherical study of the atmospheric angular momentum balance. Quart J. Roy Met. Soc., 77,215-225.

CHAPTER 2

The Zonal Momentum Balance

The equation that governs the local time rate of change of zonal wind can be written in the form

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - -v\frac{\partial u}{\partial y} - -\omega\frac{\partial u}{\partial p} + \frac{uv\tan\phi}{R_E} - \frac{\partial \Phi}{\partial x} + fv + F_x \tag{2.1}$$

A complete derivation of this equation is given in Holton (1972) p. 21-28[1]. The advective terms can be rewritten in the form

$$-u\frac{\partial u}{\partial x} - -v\frac{\partial u}{\partial y} - -\omega\frac{\partial u}{\partial p} = -\frac{\partial}{\partial x}u^2 - \frac{1}{\cos\phi}\frac{\partial}{\partial y}uv\cos\phi - \frac{\partial}{\partial p}u\omega + u\left(\frac{\partial u}{\partial x} + \frac{1}{\cos\phi}\frac{\partial}{\partial y}v\cos\phi + \frac{\partial \omega}{\partial p}\right)$$

where the term in parentheses vanishes because of the continuity of mass. Substituting back into (2.1) and making use of the identity

$$\frac{1}{\cos^2\phi}\frac{\partial}{\partial y}uv\cos^2\phi = \frac{1}{\cos\phi}\frac{\partial}{\partial y}uv\cos\phi - \frac{uv\tan\phi}{R_E}$$

we obtain

$$\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}u^2 - \frac{1}{\cos^2\phi}\frac{\partial}{\partial y}uv\cos^2\phi - \frac{\partial}{\partial p}u\omega - \frac{\partial \Phi}{\partial x} + fv + F_x \tag{2.2}$$

When we zonally average, the terms $-\partial/\partial x(u^2)$ and $-\partial\Phi/\partial x$ drop out because of the identity

$$\left[\frac{\partial \Phi}{\partial x}\right] = \frac{1}{R_E\cos\phi}\oint\frac{\partial \Phi}{\partial x}dx = 0 \tag{2.3}$$

Next we expand the $[uv]$ and $[u\omega]$ terms, making use of (1.11), to obtain

(2.4)

$$\frac{\partial [u]}{\partial t} = -\frac{1}{\cos^2\phi}\frac{\partial}{\partial y}[u][v]\cos^2\phi - \frac{\partial}{\partial p}[u][\omega] - \frac{1}{\cos^2\phi}\frac{\partial}{\partial y}[u^*v^*]\cos^2\phi - \frac{\partial}{\partial p}[u^*\omega^*] + [F_x]$$

Then we expand the mean meridional motion terms in the form

$$-\frac{1}{\cos^2\phi}\frac{\partial}{\partial y}[u][v]\cos^2\phi = -\frac{[u]}{\cos\phi}\frac{\partial}{\partial y}[v]\cos\phi - \frac{[v]}{\cos\phi}\frac{\partial}{\partial y}[u]\cos\phi$$

and

$$-\frac{\partial}{\partial p}[u][\omega] = -[u]\frac{\partial}{\partial p}[\omega] - [\omega]\frac{\partial}{\partial p}[u]$$

[1]Here the equation is written in pressure coordinates and $u\omega\tan\phi/R_E$ and the pressure coordinate form of $2\Omega w\cos\phi$ have been neglected because they are are at least two orders of magnitude smaller than the corresponding terms involving v.

Substituting back into (2.4) and making use of the zonally averaged continuity equation in spherical coordinates

$$\frac{1}{\cos\phi}\frac{\partial}{\partial y}[v]\cos\phi + \frac{\partial}{\partial p}[\omega] = 0 \tag{2.5}$$

we obtain, after some minor rearranging,

$$\frac{\partial u}{\partial t} = \left(f - \frac{1}{\cos\phi}\frac{\partial}{\partial y}[v]\cos\phi\right)[v] - [\omega]\frac{\partial [u]}{\partial p} - \frac{1}{\cos^2\phi}\frac{\partial}{\partial y}[u^* v^*]\cos^2\phi - \frac{\partial}{\partial p}[u^*\omega^*] + F_x \tag{2.6}$$

As an alternative method of deriving (2.2), we can start with the equation governing the angular momentum of a fixed, zonally symmetric annulus, bounded by latitudinal "walls" at y and $y + \delta y$ and pressure levels p and $p + \delta p$, as shown in Fig. 2.1.[2] The only processes capable of changing the integrated angular momentum within the annulus are advection across the boundaries of the annulus and frictional torques acting within the annulus. Such torques will be assumed to be small unless the annulus is contiguous with the earth's surface. The net increase in angular momentum per unit mass M due to advection across the latitudinal walls is given by

$$\int\int_y Mv dx dp - \int\int_{y+\delta y} Mv dx dp$$

where the zonal integration is carried out around a complete latitude circle and the vertical integration is carried out from level p down to $p + \delta p$. Expanding Mv in a Taylor series expansion in y, and keeping only the linear term, the above expression can be rewritten as

$$\left\{-\int\int \frac{\partial Mv}{\partial y} dx dp\right\}\delta y$$

which is an accurate representation, provided that y is sufficiently small. Furthermore, if p is sufficiently small, this expression can be vertically integrated to obtain

$$\left\{-\int\int \frac{\partial Mv}{\partial y} dx\right\}\delta y \delta p$$

or, using (1.2)

$$-2\pi R_E \delta y \delta p \frac{\partial}{\partial y}[Mv]\cos\phi$$

In a similar manner, the net increase due to vertical advection across the pressure surfaces is given by

$$-2\pi R_E \cos\phi \delta y \delta p \frac{\partial}{\partial p}[M\omega]$$

The angular momentum balance for the annulus is given by

$$\frac{\partial}{\partial t}\int\int\int M dx dy dp = -2\pi R_E \delta y \delta p \left\{\frac{\partial}{\partial y}[Mv]\cos\phi + \cos\phi \frac{\partial}{\partial p}[M\omega]\right\} + \int\int\int F_x R_E \cos\phi \tag{2.7}$$

[2] By using pressure as a vertical coordinate we are implicitly neglecting the divergence associated with $\partial/\partial r$ in the spherical coordinate system that we are using. It is this effect that leads to the small term $u\omega \tan\phi/R_E$ in the zonal momentum equation that is neglected in (2.1), as indicated in the previous footnote.

Integrating over y and p, using (1.2), and dividing through by $2\pi R_E \cos\phi \delta y \delta p$ yields

$$\frac{\partial [M]}{\partial t} = -\frac{1}{\cos\phi}\frac{\partial}{\partial y}[Mv]\cos\phi - \frac{\partial}{\partial p}[M\omega] + [F_x]\, R_E \cos\phi \tag{2.8}$$

Repeating the same set of operations that were used in transforming (2.4) into (2.6), we obtain

(2.9)

$$\frac{\partial M}{\partial t} = -[v]\frac{\partial [M]}{\partial y} - [\omega]\frac{\partial [M]}{\partial p} - \frac{1}{\cos\phi}\frac{\partial}{\partial y}[M^* v^*]\cos\phi - \frac{\partial}{\partial p}[M^*\omega^*] + [F_x]\, R_E \cos\phi$$

If we substitute

$$\frac{\partial [M]}{\partial t} = R_E \cos\phi \frac{\partial [u]}{\partial t}$$

$$\frac{\partial [M]}{\partial y} = R_E \cos\phi \left(f - \frac{1}{\cos\phi}\frac{\partial}{\partial y}[u]\cos\phi \right)$$

and

$$\frac{\partial [M]}{\partial p} = R_E \cos\phi \frac{\partial [u]}{\partial p}$$

into (2.9) and divide through by $R_E \cos\phi$ we obtain an expression identical to (2.6).

CHAPTER 3

The total energy balance

1. The zonally averaged thermodynamic energy equation

We begin by writing the First Law of Thermodynamics, as applied to a unit mass of air, in the form

$$dq = c_p dT - \alpha dp \tag{3.1}$$

and considering the incremental change over an infinitesimal time interval dt, which yields

$$\frac{dT}{dt} = \frac{\alpha}{c_p}\omega + \frac{Q}{c_p} \tag{3.2}$$

The first term on the right-hand side of (3.2) represents the rate of change of temperature due to adiabatic expansion or compression. A typical value of this term in °C per day is given by $\kappa T \delta p / p_m$, where $\delta p = \omega \delta t$ is a typical pressure change over the course of a day following an air parcel and p_m is the mean pressure level along the trajectory. In a typical middle-latitude disturbance, air parcels in the middle troposphere (p_m ~500 hPa) undergo vertical displacements on the order of 100 hPa day^{-1}. Assuming T 250 K, the resulting adiabatic temperature change is on the order of 15 °C per day.

The second term on the right-hand side of (3.2) represents the effects of diabatic heat sources and sinks: absorption of solar radiation, absorption andemission of longwave radiation, latent heat release, and, in the upper atmosphere, heat absorbed or liberated in chemical and photochemical reactions. In addition, it is customary to include, as a part of the diabatic heating, the heat added to or removed from the parcel through the exchange of mass between the parcel and its environment due to unresolved scales of motion such as convective plumes. Throughout most of the troposphere there tends to be a considerable amount of cancellation between the various radiative terms so that the net radiative heating rates are less than 1 °C per day. Latent heat release tends to be concentrated in small regions in which it may be locally comparable in magnitude to the adiabatic temperature changes discussed earlier. The convective heating within the mixed layer can also be locally quite intense, e.g., where cold air blows over much warmer ocean water. However, throughout most of the troposphere, the sum of the diabatic heating terms in (3.2) is much smaller than the adiabatic temperature change term.

Expanding the total derivative and substituting $\alpha = RT/p$ from the equation of state we obtain

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - \omega\left(\frac{\partial T}{\partial p} - \frac{\kappa T}{p}\right) + \frac{Q}{c_p} \tag{3.3}$$

where $\kappa = R/c_p$. The first two terms on the right-hand side of (3.3) represent the horizontal advection, and the third term is the combined effect of adiabatic compression and vertical advection. When the observed lapse rate is equal to the dry adiabatic lapse rate, the term in parentheses in (3.3) vanishes. In a stably stratified atmosphere, $-\partial T/\partial p$ must be less than in the adiabatic lapse rate, and thus the term in parentheses must be positive. It follows that sinking motion (or subsidence) always favors local warming and vice versa: the more stable the lapse rate, the larger the local rate of temperature increase that results from a given rate of subsidence.

Making use of the three-dimensional continuity equation, as was done in transforming (2.1) into (2.2), the advective terms in (3.3) can be expressed in flux form

$$\frac{\partial T}{\partial t} = -\frac{\partial}{\partial x}uT - \frac{1}{\cos\phi}\frac{\partial}{\partial y}vT\cos\phi - \frac{\partial}{\partial p}\omega T + \frac{\kappa T}{p}\omega + \frac{Q}{c_p} \tag{3.4}$$

When we take the zonal average, the first term on the right hand side drops out. Expanding the remaining terms into contributions from mean meridional motions and eddies yields

$$\frac{\partial [T]}{\partial t} = -\frac{1}{\cos\phi}\frac{\partial}{\partial y}[v][T]\cos\phi - \frac{1}{\cos\phi}\frac{\partial}{\partial y}[v^*T^*]\cos\phi - \frac{\partial}{\partial p}[\omega][T] - \frac{\partial}{\partial p}[\omega^*T^*] + \frac{\kappa [T]}{p}[\omega] + \frac{\kappa}{p}[\omega^*T^*] + \frac{[Q]}{c_p} \tag{3.5}$$

Using the zonally-averaged, two dimensional continuity equation, as was done in obtaining (2.6), we can convert the terms involving mean meridional circulations back to advective form

$$\frac{\partial [T]}{\partial t} = -[v]\frac{\partial [T]}{\partial y} - [\omega]\left(\frac{\partial [T]}{\partial p} - \frac{\kappa [T]}{p}\right) - \frac{1}{\cos\phi}\frac{\partial}{\partial y}[v^*T^*]\cos\phi - \frac{\partial}{\partial p}[\omega^*T^*] + \frac{\kappa}{p}[\omega^*T^*] + \frac{[Q]}{c_p} \tag{3.6}$$

On the basis of scale analysis, it can be shown that to first order

$$\frac{\partial [T]}{\partial t} \simeq s[\omega] + P + \frac{[Q]}{c_p} \tag{3.7}$$

where

$$s \equiv \left(\frac{\partial [T]}{\partial p} - \frac{\kappa [T]}{p}\right) \tag{3.8}$$

and

$$P \equiv -\frac{1}{\cos\phi}\frac{\partial}{\partial y}[v^*T^*]\cos\phi \tag{3.9}$$

2. Transport of moist static energy

The total energy per unit mass of an air parcel is given by $I + P + K$, where $I = c_v T$ is the internal energy, $P = \Phi$ is the potential energy, and $K = \frac{1}{2}(u^2 + v^2)$ is the kinetic energy. In estimating the transports of energy within the atmosphere, the transport of kinetic energy can be neglected provided that the velocities is much smaller than the speed of sound $(\gamma RT)^{1/2}$,where $\gamma = c_p/c_v = 1.4$.

Consider flow across a plane surface that might have any orientation. The mass-weight flux of internal energy is given by

$$\rho c_v T c_n dS$$

where c_n is the velocity component normal to the surface and dS is an alamant of area on the surface. The work done by the part of the atmosphere upstream of the surface on the part of the atmosphere downstream of the surface is

$$pc_n dS$$

or (substituting from the equation of state)

$$\rho RT c_n dS$$

Making use of the identity $c_v + R = c_p$, the internal energy flux and work term can be combined into the single term

$$\rho c_p T c_n dS$$

Hence, in tracking the meridional or vertical flux of energy in the general circulation, sensible heat or enthalpy $c_p T$ is used in place of internal energy.

It is convenient to include the latent heat as part of the total energy that air parcels carry with them as they move through the atmosphere. The latent heat term is given by Lq, where L is the latent heat of vaporization (2.5×10^6 J kg^{-1} K^{-1} at 0 C) and q is the specific humidity (i.e., the mass of water vapor per unit mass of air, including the water vapor, expressed as a dimensionless ratio). In practice it is sometimes convenient to represent the specific humidity in units of g/kg, in which case it should be multiplied by $L = 2500$ J g^{-1} K^{-1}.

CHAPTER 4

The hydrologic cycle

1. The atmospheric branch

For a vertical column of the atmosphere we can write

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{Q} = E - P \tag{4.1}$$

where W is the mass of water vapor in the column,

$$\mathbf{Q} = \frac{1}{g} \int_0^{p_0} q \mathbf{V} dp \tag{4.2}$$

is the vertically-integrated water vapor flux vector, q is (dimensionless) specific humidity, E is evaporation and P is precipitation, which are assumed to be the only sources and sinks of water vapor. We ignore the storage of water in the form of cloud droplets, rain drops, and ice particles.because cloud liquid (and solid) water content is generally much less than the density of water in the vapor form except in deep convective clouds, which cover only a snall fraction of the area of the Earth. The time (e.g., climatological- or seasonal-) mean vertically-integrated water vapor flux can be decomposed into contributions from the time mean fields and the transient variability within the averaging period; i.e.,

$$\overline{\mathbf{Q}} = \mathbf{Q}_M + \mathbf{Q}_T \tag{4.3}$$

where

$$\mathbf{Q}_M = \frac{1}{g} \int_0^{p_0} \overline{q} \overline{\mathbf{V}} dp$$

and

$$\mathbf{Q}_T = \frac{1}{g} \int_0^{p_0} \overline{q' \mathbf{V}'} dp$$

Averaged over periods of a few days or longer, the time rate of change of W is negligible compared to the other terms in (4.1) so that this equation reduces to the balance requirement

$$\nabla \cdot \overline{\mathbf{Q}} = \overline{E} - \overline{P} \tag{4.4}$$

where overbars denote time averages. In the zonal average

$$\frac{1}{\cos \phi} \frac{\partial}{\partial y} \left[\overline{\mathbf{Q}} \right] \cos \phi = \left[\overline{E} \right] - \left[\overline{P} \right] \tag{4.5}$$

where the zonally-averaged, meridional water vapor transport $[Q]$ can be decomposed into contributions from mean meridional circulations and eddies, and time mean and transuent contributions, as explained in Appendix 1.

The distribution of P has much more horizontal structure than that of E and in areas of heavy rainfall such as the monsoons and the ITCZ, $P >> E$. It follows that in these regions, most of the water that falls as rain is imported by way of the moisture flux convergence term. Water vapor is exported out of regions where $E > P$, such as the subtropical dry zones, and into regions of heavy rainfall such as the ITCZ, the monsoons and the midlatitude storm tracks.

Most of the vertically-integrated water vapor transport occurs within the boundary layer. Hence,

$$\mathbf{Q} \sim \widehat{q}\widehat{\mathbf{V}}\frac{\delta p}{g}$$

where $\widehat{q}$ and $\widehat{\mathbf{V}}$ represent vertical averages through the boundary layer and δp is a typical boundary layer depth.This expression may be simplified further by usin the identity

$$\nabla \cdot q\mathbf{V} = q\nabla \cdot \mathbf{V} + \mathbf{V} \cdot \nabla q$$

and noting that the first term tends to be dominant except sometimes near fronts because $\nabla \cdot \mathbf{V}/\mathbf{V} >> \nabla q/q$. Hence,

$$\nabla \cdot \mathbf{Q} \sim \nabla \cdot \mathbf{V}\left(\frac{q_0 \delta p}{g}\right)$$

where q_0 is a reference value of q, for example, a typical value in the tropical boundary layer. The qualiiative similarity between the spatial patterns in the fields of $\nabla \cdot \mathbf{Q}$ and $\nabla \cdot \mathbf{V}$ explains why heavy rain belts such as the ITCZ correspond closely with bands of low level convergence.

2. The land surface branch

The analog of (3.1) for the land surface is

$$P - E = \frac{\partial}{\partial t}Storage + R \tag{4.7}$$

where R is the runoff in rivers and subsurface aquifers. Rather than being evaluated locally, the balance in (3.6) is usually averaged over a regious such as a river valley, bounded either by divide, across which $R = 0$, or by a coastline. Storage reservoirs include lakes and ground water. The time rate of storage exhibits large seasonal variations in response to seasonal variations in precipitation and the tendency for increased evaporation during the warm season. Combining (4.1) and (4.7), taking a time average long enough to ensure that the $\partial W/\partial t$ term in (4.1) is negligible yields

$$\frac{\partial}{\partial t}Storage + R = -\nabla \cdot \mathbf{Q} \tag{4.8}$$

which shows how the atmospheric water vapor budget drive the land hydrology. For the special case of a land-locked drainage basin, such as the Great Basin in the interior of the western United States, R is identically equal to zero and (4.8) reduces to

$$\frac{\partial}{\partial t}Storage = -\nabla \cdot \mathbf{Q} = P - E \tag{4.9}$$

In a land-locked basin, low frequency variations in precipitation can give rise to large variations in storage, which may be manifested in rising or falling lake levels. As the storage increases, the evaporation increases in response to the increasing areal coverage of lakes. If it is assumed that E is directly proportional to storage, the, (4.9) can be written

$$\frac{\partial}{\partial t} Storage = P - k(Storage) \tag{4.10}$$

in which case, the storage is perturbed by variations in precipitation, analogous to Brownian motion, but the linear damping term serves as a negative feedback that tends to draw the system back toward equilibrium